## D2 Paper F - Marking Guide

1. e.g. using stage, state approach:

| Stage | State | Destination | Cost | Total cost |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Marquee | Deluxe | 20 | $20^{*}$ |
|  |  | Cuisine | 24 | 24 |
|  | Castle | Deluxe | 21 | 21 |
|  |  | Castle | 15 | $15^{*}$ |
|  |  | Cuisine | 22 | 22 |
|  | Hotel | Deluxe | 18 | $18^{*}$ |
|  |  | Cuisine | 23 | 23 |
|  |  | Hotel | 19 | 19 |
| 2 |  | Marquee | 2 | $2+20=22$ |
|  |  | Castle | 5.5 | $5.5+15=20.5^{*}$ |
|  |  | Hotel | 3 | $3+18=21$ |
|  | Castle | Marquee | 3 | $3+20=23$ |
|  |  | Castle | 5 | $5+15=20^{*}$ |
|  | Registry | Marquee | 3.5 | $3.5+20=23.5$ |
|  | Office | Castle | 6 | $6+15=21$ |
|  |  | Hotel | 2 | $2+18=20^{*}$ |
| 3 | Home | Castle | 3 | $3+20.5=23.5$ |
|  |  | Church | 5 | $5+20=25$ |
|  |  | Registry | 1 | $1+20=21^{*}$ |

M1 A1

M1 A2

A1
minimum cost $£ 2100$ using
ceremony - Registry Office, reception - Hotel, catering - Deluxe
A1
(7)
2. (a) let $A$ play strategies I and II with proportions $p$ and $(1-p)$ expected payoff to $A$ against each of $B$ 's strategies:
$B$ I $\quad p+3(1-p)=3-2 p$
$B$ II $\quad-p+5(1-p)=5-6 p$
M1 A1
$B$ III $2 p-(1-p)=3 p-1$
giving


M1 A1
it is not worth player $B$ considering strategy I
(b) for optimal strategy $5-6 p=3 p-1$ M1

$$
\therefore 9 p=6, p=\frac{2}{3}
$$

$\therefore A$ should play I $\frac{2}{3}$ of time and II $\frac{1}{3}$ of time
value of game $=5-\left(6 \times \frac{2}{3}\right)=1$
A1
A1
3.
row min.

| 5 | 20 | 12 | 18 | 5 |
| :--- | :--- | :--- | :--- | :--- |
| 6 | 18 | 15 | 16 | 6 |
| 4 | 21 | 9 | 15 | 4 |
| 5 | 16 | 11 | 13 | 5 |

reducing rows gives:
$\begin{array}{llll}0 & 15 & 7 & 13\end{array}$
$\begin{array}{llll}0 & 12 & 9 & 10\end{array}$
$\begin{array}{llll}0 & 17 & 5 & 11\end{array}$
01168
col min.
$\begin{array}{lll}0 & 11 & 5\end{array}$
reducing columns gives:
$\begin{array}{llll}0 & 4 & 2 & 5\end{array}$
$\begin{array}{llll}0 & 1 & 4 & 2\end{array}$
$\begin{array}{lllll}-0 & 6 & 0 & 3 & \text { (N.B. a different choice of lines will }\end{array}$
A1
lead to the same final assignment)
3 lines required to cover all zeros, apply algorithm
B1

$$
\begin{array}{llll}
0^{*} & 3 & 1 & 4 \\
0 & 0^{*} & 3 & 1 \\
1 & 6 & 0^{*} & 3 \\
2 & 0 & 1 & 0^{*}
\end{array}
$$

4 lines required to cover all zeros so allocation is possible
Andrew reviews a film
Betty reviews a musical
Carlos reviews a ballet
Davina reviews a concert
total cost $=5+18+9+13=£ 45$
A1
4. (a) $x=2, y=14$

B2
(b) (i), (ii) e.g. augment $S C T$ by 2 and SBECADT by 3 giving:

(c) (i) minimum cut $=53$, passing through $D T, C T$ and $E T$
(ii) max flow $=$ min cut
it is not possible to get any more flow across this cut

B1
B1 (9)
5. (a) maximise $R=10 x+12 y+8 z$ given

$$
\begin{aligned}
& x+2 y+4 z \leq 20 \\
& 4 x+3 y+14 z \leq 75 \\
& 5 x+2 y+10 z \leq 60 \\
& x \geq 0, \quad y \geq 0, \quad z \geq 0
\end{aligned}
$$

(b) to change inequalities into equations
(c) $\theta$ values are 10,25 and 30 so pivot row is $2^{\text {nd }}$ row
$2^{\text {nd }}$ tableau is:

| $R$ | $x$ | $y$ | $z$ | $s$ | $t$ | $u$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -4 | 0 | 16 | 6 | 0 | 0 | 120 |
| 0 | $\frac{1}{2}$ | 1 | 2 | $\frac{1}{2}$ | 0 | 0 | 10 |
| 0 | $\frac{5}{2}$ | 0 | 8 | $-\frac{3}{2}$ | 1 | 0 | 45 |
| 0 | 4 | 0 | 6 | -1 | 0 | 1 | 40 |

M2 A2
choose to increase $x$ next
$\theta$ values are 20,18 and 10 so pivot row is $4^{\text {th }}$ row
$3^{\text {rd }}$ tableau is:

| $R$ | $x$ | $y$ | $z$ | $s$ | $t$ | $u$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 22 | 5 | 0 | 1 | 160 |
| 0 | 0 | 1 | $\frac{5}{4}$ | $\frac{5}{8}$ | 0 | $-\frac{1}{8}$ | 5 |
| 0 | 0 | 0 | $\frac{17}{4}$ | $-\frac{7}{8}$ | 1 | $-\frac{5}{8}$ | 20 |
| 0 | 1 | 0 | $\frac{3}{2}$ | $-\frac{1}{4}$ | 0 | $\frac{1}{4}$ | 10 |

M1 A2
(d) optimal solution as all values on the objective row are $\geq 0$

B1
company donates 10 two-person and 5 four-person boats
B1
(12)
6. (a)

(b) lower figures give forward scan upper figures give backward scan
critical path is ACDEHJK
minimum time is 41 hours
(c)

(d) no. of workers

$\therefore$ max. of 15 workers required
M1 A2
(e) no. of workers


M1 A1
(15)

